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Nielsen-Olesen strings in supersymmetric models

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Abstract: We investigate the behaviour of a model with two oppositely charged scalar fields. In the Bogomol'nyi limit this may be seen as the scalar sector of $N = 1$ supersymmetric QED, and it has been shown that cosmic strings form. We examine numerically the model out of the Bogomol'nyi limit, and show that this remains the case. We then add supersymmetry-breaking mass terms to the supersymmetric model, and show that strings still survive. Finally we consider the extension to $N = 2$ supersymmetry with supersymmetry-breaking mass terms, and show that this leads to the formation of stable cosmic strings, unlike in the unbroken case.

Keywords: Solitons Monopoles and Instantons, Spontaneous Symmetry Breaking, Supersymmetric Effective Theories, Cosmological Phase Transitions.

Contents

1. Introduction

Supersymmetric (SUSY) theories often have a continuous degeneracy of inequivalent ground states. Classically this corresponds to flat directions of the scalar potential, which may arise from abelian theories when the gauge symmetry is broken by a Fayet-Iliopoulos D-term. Recently, these have undergone intense study, mostly in the context of effective actions for SUSY non-abelian gauge theories and confinement [1].

We naturally expect the formation of cosmic strings when the D-term breaks the gauge symmetry [2, 3], but the presence of the flat directions means that the situation is more complicated. It is not immediately obvious which vacuum states among the degenerate vacua will give rise to stable string solutions.

The situation has been investigated in [2] where a model with two oppositely charged chiral multiplets and zero superpotential is discussed. They show that one particular vacuum is favoured, and that a Nielsen-Olesen (NO) [4] type cosmic string can form in one of the scalar fields, the other field being zero everywhere. It was later shown in [5] that the crucial role in this vacuum selection effect is that played by the gauge field. The vaccum is effectively chosen so that the vector mass is minimized, leading to a lower energy for the core of the vortex. The authors in [5] also proved that the preferred cosmic string was stable, using a Bogomol'nyi type of argument.

In this paper we consider the scalar sector of this theory outside the Bogomol'nyi bound to study whether the result mentioned above holds as well for other values of the parameters. Although it no longer corresponds to the simplest SUSY model, it still possesses flat directions, and may be viewed as illustrating the behaviour of more general SUSY theories with flat directions. An analytical approach is not possible out of the Bogomol'nyi bound, so we carry out numerical simulations of the fields to see if there is still any evidence

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of vacuum selection occurring. In fact the simulations suggest that, for all the range of parameters considered, the system chooses one out of all the possible minima of the flat directions and forms NO strings.

We then study the fate of the different defects formed in the $N = 1$ and $N = 2$ supersymmetric QED models considered in [5] in which we break supersymmetry by the addition of soft supersymmetry-breaking mass terms to the scalars. Then, the degeneracy in the minima of the potential is lifted, and we have no flat directions. For the $N = 1$ case, the NO string which is present without mass terms can survive SUSY breaking for some values of the masses.

The situation is different for the $N = 2$ model. The system was first considered in [6], in which it was claimed that the model accommodated strings that could confine magnetic charges. However, [7] showed that the strings were not stable, and so could not confine. A further step attemping to get stable strings by adding a Fayet-Iliopoulos term [5] proved to be insufficient. Although there is a vacuum selection effect in the model, it is not enough to form *stable* cosmic strings. The strings formed are semilocal [8], and are only neutrally stable, in the sense that small perturbations tend to cause the string core to expand indefinitely [9]. We will show that with the additional supersymmetry breaking mass terms, stable cosmic strings are possible.

2. A scalar model out of the Bogomol'nyi bound

The model we consider has two complex scalar fields ϕ_+ and ϕ_- of opposite charges, coupled to a U(1) vector field. The lagrangian is of the form

$$\mathcal{L} = |D_\mu\phi_+|^2 + |D_\mu\phi_-|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}\left(|\phi_+|^2 - |\phi_-|^2 - \eta^2\right)^2 , \tag{2.1}$$

where A_μ is the U(1) gauge field and, as usual,

$$\begin{aligned} D_\mu\phi_+ &= (\partial_\mu + iqA_\mu)\,\phi_+ ; \\ D_\mu\phi_- &= (\partial_\mu - iqA_\mu)\,\phi_- ; \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu . \end{aligned} \tag{2.2}$$

The above can be regarded, in the Bogomol'nyi limit $\lambda = q^2$, as the bosonic part of a four-dimensional model with two $N = 1$ chiral superfields (Φ_+, Φ_-) of opposite charges $\pm q$ coupled to a U(1) vector multiplet, with zero superpotential and a Fayet-Iliopoulos term proportional to η^2.

The vacuum manifold of this model is given by

$$|\phi_+|^2 - |\phi_-|^2 = \eta^2 , \tag{2.3}$$

with solutions

$$|\phi_+| = \eta\cosh u \equiv v_+ , \qquad |\phi_-| = \eta\sinh u \equiv v_- , \tag{2.4}$$

where u parametrises the moduli space. When $u = 0$, the field ϕ_- is zero, and formation of the usual NO string [4] in the field ϕ_+ may occur, since we are back at the abelian Higgs model.

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By considering the energy per unit length of straight static vortices, we find that the conditions for finite energy impose some relations between the fields: since we are looking to construct cylindrically symmetric configurations, fields with charge $\pm q$ wind as $e^{\pm iqn\theta}$, and the gauge field tends to a constant $A_\theta \to -n$. Bearing this in mind, we use the ansatz

$$\phi_+ = f_+(r)e^{iqn\theta}\,; \qquad \phi_- = f_-(r)e^{-iqn\theta}e^{i\Delta}\,; \qquad A_\theta \mathrm{d}\theta = a(r)\mathrm{d}\theta\,, \tag{2.5}$$

where $f_\pm$ and a are real functions, and Δ is a real constant. $f_\pm(0) = a(0) = 0$ for the functions to be well-defined, and $a(r \to \infty) = -n$ by the previous energy arguments. Such considerations also tell us that the scalar fields should go to a minimum of the potential energy as $r \to \infty$, and hence the boundary conditions for the $f_\pm$ functions are $f_\pm(r \to \infty) = v_\pm$. It is possible to construct vortices tending to any of the vacua (2.4) as $r \to \infty$.

However, it has been shown [2, 5] that when the Bogomol'nyi limit of this system ($\beta = \lambda/q^2 = 1$) is considered, the only static solutions are those with $u = 0$. After symmetry breaking, apart from the massless fields, we are left with a massive vector field ($m_v^2 = 2q^2\eta^2 \cosh 2u$) and a massive scalar field ($m_s^2 = 2\lambda\eta^2 \cosh 2u$). The system chooses $u = 0$ in order to minimize the vector mass.

Thus NO strings may form only in the ϕ_+ field, since $f_-(r) = 0$. In the following section we would like to move out of the Bogomol'nyi limit to see whether $u = 0$ is still preferred by the system. In fact, as explained in [2], the dynamics of the magnetic core and those of the scalar fields outside the core are effectively decoupled. Far from the core, the magnetic field is very small, and the scalar fields lie on the vacuum manifold. Hence, scalar fields are free to move along the moduli space with no appreciable cost in energy, so one might presume that some ϕ_- lumps would be formed far from the core of the string and this might have some cosmological consequences.

3. Numerical simulations

We use numerical simulations of a discretized version of the model described by (2.1) in order to study its behaviour out of the Bogomol'nyi bound. We firstly set $\eta = q = 1$ by rescaling the fields and spacetime coordinates, and then we use techniques borrowed from hamiltonian lattice gauge theory (see for instance [10]) to perform the simulations.

The lattice link and plaquette operators are defined as

$$U_i(x) = e^{-ilA_i(x)}\,; \tag{3.1}$$

$$Q_{ij} = U_j(x)U_i(x+x_j)U_j^\dagger(x+x_i)U_i^\dagger(x)\,, \tag{3.2}$$

respectively, where l is the lattice spacing, the label i takes the values $1, 2, 3$ corresponding to the three spatial dimensions, and A_i are the gauge fields. By $x + x_i$, we denote the nearest lattice point in the i direction from x. The plaquette operators are related to the gauge field strength. The lattice link operator, on the other hand, is used to define discrete covariant derivatives

$$\begin{aligned} D_i\phi_+(x) &= \frac{1}{l}\left(U_i^\dagger(x)\ \phi_+(x+x_i) - \phi_+(x)\right); \\ D_i\phi_-(x) &= \frac{1}{l}\left(U_i(x)\phi_-(x+x_i) - \phi_-(x)\right). \end{aligned} \tag{3.3}$$

We can then obtain a discretized hamiltonian density corresponding to the lagrangian (2.1)

$$\mathcal{H} = |\Pi_+|^2 + |\Pi_-|^2 + \frac{1}{2}E_i E^i + \frac{\beta}{2}\left(|\phi_+|^2 - |\phi_-|^2 - 1\right)^2 + \\ +|D_i\phi_+|^2 + |D_i\phi_-|^2 + \frac{1}{2l^4}\sum_{i\neq j}\left(1 - \mathrm{Re}(Q_{ij})\right) , \tag{3.4}$$

where $\Pi_\pm$ and E^i are the conjugate momenta of $\phi_\pm$ and A_i respectively, and $\beta = \lambda/q^2$ ($\beta = \lambda$ in rescaled units). In the $l \to 0$ limit, the continuum hamiltonian for the system is recovered.

One interesting property of (3.4) is that it is gauge invariant, in the sense that a general U(1) transformation $\Lambda(x)$ for which the fields transform as

$$\begin{aligned} \phi_+(x) &\to \Lambda(x)^\dagger \phi_+(x) ; \\ \phi_-(x) &\to \Lambda(x)\phi_-(x) ; \\ U_i(x) &\to \Lambda(x) U_i(x) \Lambda^\dagger(x + x_i) , \end{aligned} \tag{3.5}$$

leaves the hamiltonian invariant.

In order to get the hamiltonian in the form (3.4) a gauge choice has been made, namely, $A_0 = 0$. The equation of motion corresponding to A_0 is equivalent to Gauss's law

$$il\left(\phi_+^\dagger(x)\Pi_+ - \Pi_+^\dagger\phi_+(x) + \Pi_-^\dagger(x)\phi_-(x) - \phi_-^\dagger(x)\Pi_-(x)\right) = \sum_i\left(E^i(x) - E^i(x - x_i)\right) . \tag{3.6}$$

The system is geometrically forced to preserve Gauss's law, and, if the initial conditions satisfy relation (3.6), subsequent evolution by the equations of motion coming from (3.4) will also satisfy (3.6).

The gauge invariant dissipative terms $\gamma\Pi_\pm$ and γE^i were added to the hamiltonian equations of motion for $\phi_\pm$ and A_i respectively. These obey Gauss's law, and thus the system as a whole will still obey Gauss's law as well. γ is a free parameter in the model, but simulations using different values of γ show qualitatively similar behaviour.

The equations of motion coming from the hamiltonian (3.4) were simulated numerically on a 64^3 lattice. We are interested in the role played by the ϕ_- field in the dynamics of the system, not in the details of the phase transition itself. Furthermore, previous work concerning the numerical simulation of defects showed that the formation of defects depends on the later interplay between scalar and gauge fields, and is not actually very sensitive to the initial configuration used [11, 12]. We did, however, use different initial conditions to verify that the result was not dependent on how the system was set. In fact, all simulations using different initial conditions give rise to qualitatively the same results. The first time steps are a transient period during which the system loses energy rapidly, and relaxes. Then, the interaction between gauge and scalar fields leads to the formation of defects.

All the results shown in this paper are obtained by using $\phi_\pm = 0$, $A_\mu = 0$ and $\dot{A}_\mu = 0$ and random initial velocities for the scalar fields as the initial configuration. This choice of conditions obeys Gauss's law, and hence we have guaranteed that Gauss's law holds throughout the simulation. Computation of Gauss's law during the evolution of different initial conditions was used to ensure the stability of the code.

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4. Numerical results

Figure 1 is a schematic example of the outcome of the simulations. It can be seen that there exists a transitional period during the early timesteps, when the system is trying to get a physical configuration by dissipating energy. Both scalar fields acquire non-zero values at this time, and in particular $\phi_- \neq 0$. Some of the initial kinetic energy is transformed to potential energy, but there is no hint of structure of any kind yet. At the end of the transitional period, $|\phi_-|$ decreases rapidly until $|\phi_-| \sim 0$.

In figure 1(a) we have plotted the number of sites in the lattice with $|\phi_+| < 0.75$, and also the number with $|\phi_-| > 0.1$, as a fraction of the total number of lattice sites. After the transitional period ($t \sim 15$), there are no points on the lattice with $|\phi_-| > 0.1$, whereas the field ϕ_+ has modulus greater than 0.75 almost everywhere. The scalars fields should tend to minimize the potential energy, and according to these results, it seems that only ϕ_+ is involved in achieving this. This is strongly suggestive of the idea that, out of all the possibilities, the system has chosen $u = 0$ as the preferred point in the flat directions (2.4).

In figure 1(b) we have plotted the quantity $1 - \sum_x \frac{V(x)}{V_{\max}}$, where $V(x)$ is the potential energy at lattice point x, and $V_{\max}$ is defined as the total potential energy calculated in the case when $\phi_\pm = 0$ everywhere. $\sum_x |\phi_+(x)|$ and $\sum_x |\phi_-(x)|$ are also plotted as fractions of the total number of lattice points. In this figure we see that, as in (a), after the transitional period ϕ_- tends quickly towards zero, while $|\phi_+| \sim 1$ almost everywhere. The potential energy mimics the behaviour of ϕ_+, and ignores the presence of ϕ_-, suggesting once again that the actual minimization of the potential energy is due to the field ϕ_+ alone.

The system is not exactly at the minimum of the potential everywhere, as there are a number of points in the lattice at which $|\phi_+| < 0.75$. This indicates that there are points at the false vacuum ($\phi_+ = \phi_- = 0$), and there may be strings forming in the system. In

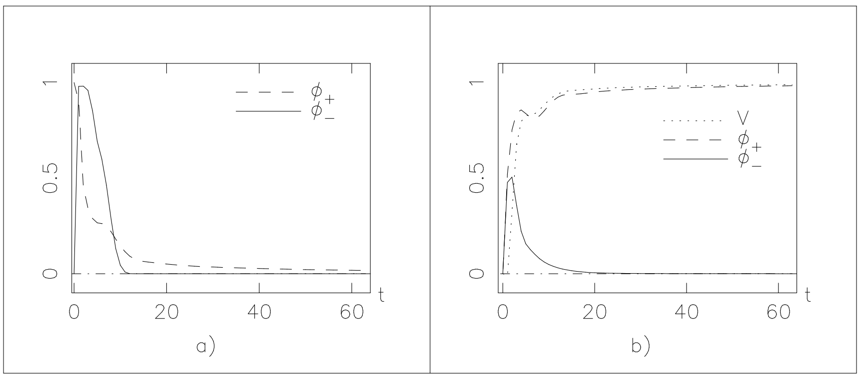

Figure 1: Representation of the results of a numerical simulation performed in a 64^3 lattice, with $\beta = 0.3$ and $\gamma = 0.5$. Figure (a) shows the number of lattice sites with $|\phi_+| < 0.75$ (dashed line) and $|\phi_-| > 0.1$ (continuous line) as a fraction of the total number of sites. Figure (b) shows the sum of the modulus of each of the fields at every site (as a fraction of the total lattice points): $\sum_x |\phi_+|$ (dashed line), $\sum_x |\phi_-|$ (continuous line). There is also a representation of the quantity $1 - \sum_x V(x)/V_{\max}$, where $V(x)$ is the potential energy (dotted line).

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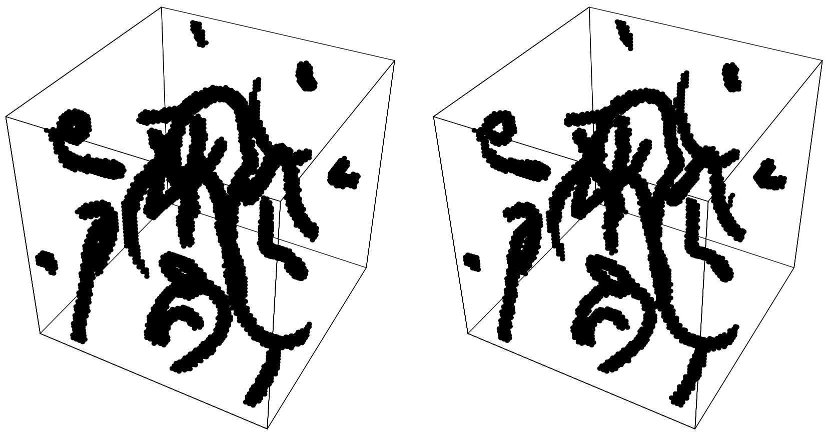

Figure 2: Isosurface representation of the modulus of ϕ_+ (left), where $|\phi_+| \sim 0.75$ is shown, and magnetic field (right), where $\sqrt{\frac{1}{2}F_{ij}F^{ij}} \sim 0.25$ of the measured maximum is displayed, for a 64^3 simulation with $\beta = 0.3$, $\gamma = 0.5$ at $t = 40$.

figure 2 we give a representation of the magnetic field, $\sqrt{\frac{1}{2}F_{ij}F^{ij}}$, and the modulus of the field ϕ_+ at time $t = 40$, after the transient period is over. It is clear that the points where $|\phi_+| < 0.75$ form structures, namely strings, and that the areas in which the magnetic field is concentrated corresponds exactly to these points. A similar plot using $|\phi_-|$ would show that $|\phi_-| \sim 0$ everywhere. The excellent match described above between the magnetic field and $|\phi_+|$ is another good check of the stability of our code. We performed several simulations varying β, from $\beta = 0.1$ to $\beta = 2.0$, which showed the same behaviour.

These results show that after some initial (non-physical) period of relaxation, only one of the scalars (ϕ_+) becomes dynamical, and that the other (ϕ_-) goes rapidly to zero everywhere, decoupling from the other fields. Thus the analytical result obtained for $\beta = 1$ [5] has been shown to be valid for cases with $\beta \neq 1$. In all cases the system chooses the vacuum with $u = 0$, and the field ϕ_- is effectively decoupled from the dynamics. After this decoupling, we are left with what essentially is an abelian Higgs model and therefore the system accommodates stable Nielsen-Olesen strings, formed by interaction of the gauge field with ϕ_+.

5. Adding mass terms

In the previous section we demonstrated that although the vacuum of the system investigated has flat directions, the only vacuum configuration appearing in the evolution of a string network is the one with $u = 0$, which is a stable cosmic string. We will now proceed to show another situation in which stable NO strings form in this context. We first add mass terms to our lagrangian (2.1), obtaining NO vortices again, and then move on to a more complicated system with $N = 2$ SUSY, which we will again break with mass terms. In this second case, when there are no mass terms, the strings formed are semilocal strings [8], which in the limit considered are neutrally stable. There is a family of such strings, all degenerate in energy, with different widths ranging from that of the NO vortex

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to an infinite core [14]. There exists a zero mode corresponding to changes in the width of the core, which when excited causes any given string in this family to grow indefinitely.

Since SUSY is clearly not manifest at low energies, it is necessary that it be broken at some scale. Adding these soft supersymmetry-breaking mass terms to the lagrangian is a convenient way of doing so while avoiding the details of how the breaking actually occurred.

The new lagrangian, modified from (2.1), is

$$\tilde{\mathcal{L}} = |D_\mu\phi_+|^2 + |D_\mu\phi_-|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}\left(|\phi_+|^2 - |\phi_-|^2 - \eta^2\right)^2 - m_+^2|\phi_+|^2 - m_-^2|\phi_-|^2\,, \tag{5.1}$$

and the two extrema of the potential are now at

$$\phi_+ = 0\,, \qquad \phi_- = 0\,, \tag{5.2}$$

and

$$|\phi_+|^2 = \eta^2 - \frac{m_+^2}{\lambda}\,, \qquad \phi_- = 0\,. \tag{5.3}$$

When $\lambda\eta^2 > m_+^2$, the second case is clearly the minimum. The degeneracy of the vacuum is lifted, $u = 0$ is selected out of the possible vacua (2.4), and we are left with a genuine NO vortex. If the condition $\lambda\eta^2 > m_+^2$ does not hold, the only solution has both fields equal to zero, and no strings form. Therefore, depending in the value of m_+, the system either accomodates a NO string solution in ϕ_+, or no string solution is possible at all. As in the previous case, ϕ_- does not play any role in the defect formation.

A more interesting result is obtained when this $N = 1$ model is upgraded to $N = 2$. We consider a model consisting of two $N = 2$ hypermultiplets h_a, where $a = 1, 2$, of opposite charges $q_a = \pm q$, coupled to an $N = 2$ abelian vector multiplet. Such a model was considered in [6] as a candidate for obtaining magnetic flux confinement in the low energy limit of type-II superstrings compactified on Calabi-Yau manifolds. The vortex solution obtained in [6] was found to be unstable [13], since the U(1) gauge symmetry was not broken. A new attempt to obtain stable vortices in this model [5] added a Fayet-Iliopoulos D-term of the form $\vec{k}\cdot\vec{D}$ to the system. This gives rise to a term that spontaneously breaks the gauge symmetry. The defects obtained in this model were the so-called semilocal strings [8], a one-parameter set of magnetic "vortices", which includes the NO vortex. In the limit studied, semilocal strings are known to spread out to greater and greater radius, as mentioned before, until the string relaxes to the vacuum [14], although the magnetic field remains quantized.

In the present work, we add soft supersymmetry-breaking mass terms to the Higgs scalars of the model studied in [5], and investigate the resulting vortices.

Without loss of generality we take $\vec{k} = (0, 0, \frac{\eta^2}{2})$. The energy of the system is

$$\begin{aligned} E = \int\! \mathrm{d}^2x\, \frac{1}{2}\Big[& |D_\mu h_{11}|^2 + |D_\mu h_{12}|^2 + |D_\mu h_{21}|^2 + |D_\mu h_{22}|^2 + B^2 + \\ & + \left(H_1^{\,1} - H_2^{\,2} + \frac{\eta^2}{2}\right)^2 + \left(H_2^{\,1} + H_1^{\,2}\right)^2 + \left(i\,H_2^{\,1} - i\,H_1^{\,2}\right)^2 + \\ & + m_{11}^2|h_{11}|^2 + m_{12}^2|h_{12}|^2 + m_{21}^2|h_{21}|^2 + m_{22}^2|h_{22}|^2 \Big]\,, \end{aligned} \tag{5.4}$$

where $D_\mu = \partial_\mu + i(q_a)A_\mu$ and $H_j^{\ i} = -(q_a/2)\, h_{ai}^* h_{aj}$.

The original model had, in addition to the U(1) gauge symmetry, an effective global SU(2) symmetry of the scalar potential, mixing the pairs of fields (h_{11} and h_{22}^*) and (h_{12} and h_{21}^*) amongst themselves. This symmetry was responsible for the appearance of semilocal strings. It would therefore be expected that the addition of different mass terms $m_{11} \neq m_{22}$ would lead to the formation of NO vortices, as the SU(2) symmetry is broken by those terms.

Parametrising the scalars as $h_{ai} = r_{ai} e^{i\chi_{ai}}$, the potential takes the form

$$\begin{aligned} V = \frac{1}{8}\bigg(& \left(-r_{11}^2 + r_{21}^2 + r_{12}^2 - r_{22}^2 + \eta^2\right)^2 + 4r_{11}^2 r_{12}^2 + 4r_{21}^2 r_{22}^2 - \\ & - 8\left(r_{11} r_{12} r_{21} r_{22} \cos(\chi_{11} + \chi_{22} - \chi_{12} - \chi_{21})\right)\bigg) + \\ & + \frac{1}{2}\left(m_{11}^2 r_{11}^2 + m_{12}^2 r_{12}^2 + m_{21}^2 r_{21}^2 + m_{22}^2 r_{22}^2\right). \end{aligned} \tag{5.5}$$

In order to obtain the minimum potential we require $\cos(\chi_{11} + \chi_{22} - \chi_{12} - \chi_{21})$ is maximised, so that $\chi_{11} + \chi_{22} - \chi_{12} - \chi_{21} = 0$ modulo 2π. The vacuum manifold can then be calculated by varying V over all the fields, $\frac{\delta V}{\delta r_{ai}} = 0$, and we find the following minima

$$\begin{aligned} r_{11}^2 &= \eta^2 - 2m_{11}^2\,, & r_{12} = r_{21} = r_{22} = 0\,; \\ r_{12}^2 &= -\eta^2 - 2m_{12}^2\,, & r_{11} = r_{21} = r_{22} = 0\,; \\ r_{21}^2 &= -\eta^2 - 2m_{21}^2\,, & r_{11} = r_{12} = r_{22} = 0\,; \\ r_{22}^2 &= \eta^2 - 2m_{22}^2\,, & r_{11} = r_{12} = r_{21} = 0\,, \end{aligned} \tag{5.6}$$

together with the maximum

$$r_{11} = r_{12} = r_{21} = r_{22} = 0\,, \tag{5.7}$$

supposing that all masses are different. We see that solutions with $r_{12} \neq 0$ and $r_{21} \neq 0$ are never valid, so we are left with only two possible minima. These two minima correspond to NO vortices in r_{11} or r_{22}, and depending on the values of m_{11} and m_{12} one or the other will form.

For $m_{11} < m_{22}$ and $2m_{11}^2 < \eta^2$ the NO strings will form in the h_{11} field; on the other hand, if $m_{22} < m_{11}$ and $2m_{22}^2 < \eta^2$ the NO will be in the h_{22} field. If the masses are bigger than η^2, then no vortex solution is possible.

A special case is when $m_{11} = m_{22} < \eta^2/2$, since in this instance the SU(2) symmetry between h_{11} and h_{22}^* is not broken, and we recover semilocal strings as in [5]. The minima in this case are

$$r_{11}^2 + r_{22}^2 = \eta^2 - 2m_{11}^2\,, \tag{5.8}$$

and we have a whole family of solutions [14], the NO string among them.

We then conclude that, depending on the values of the added masses m_{11} and m_{22}, we might have the case where no strings at all form, the case where we recover semilocal strings, or the system might be able to form strings in h_{11} or h_{22} that are stable NO vortices.

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6. Discussion

We have seen that, for a general class of scalar fields with flat directions in the potential, there does seem to be a vacuum selection effect taking place even when the system is out of the Bogomol'nyi limit. Furthermore, the vacuum chosen remains the same as in the Bogomol'nyi limit, regardless of the value of the parameter β. After spontaneous symmetry breaking, defects may form. Due to this vacuum selection effect only one of the fields becomes dynamical and this field gives rise to Nielsen-Olesen strings, whereas the other field decouples. This is what is seen in simulations.

On the other hand, for supersymmetric models with flat directions, the addition of supersymmetry-breaking mass terms, that arise in the effective SUSY action at low energies, lifts these flat directions. In the case that $m_{11} = m_{22}$, the string is a semilocal string in the Bogomol'nyi limit. This solution has the same form as that of the model without masses, where there are no stable string solutions. Nevertheless, if the masses are different, only one scalar field has a non-zero vacuum expectation value. This field then gives rise to a NO cosmic string.

In addition, in supersymmetric theories there will be fermion zero modes in the string cores due to the symmetry between bosons and fermions [3]. In the case where the supersymmetry is broken, it may still be possible to have fermionic zero modes on these strings. The zero modes give rise to currents that can stabilise string loops. We will investigate this possibility in a subsequent paper [15].

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